CHAPTER 3
MEASUREMENT AND INFERENCE

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3.1 THE MEASUREMENT PROBLEM

The essential purpose and basic function of all branches of engineering is design. Design begins with the recognition of a need and the conception of an idea to meet that need. One may then proceed to design equipment and processes of all varieties to meet the required needs. Testing and experimental design are now considered a necessary design step integrated into other rational procedures. Experimentation is often the only practical way of accomplishing some design tasks, and this requires measurement as a source of important and necessary information.

To measure any quantity of interest, information or energy must be transferred from the source of that quantity to a sensing device. The transfer of information can be accomplished only by the corresponding transfer of energy. Before a sensing device or transducer can detect the signal of interest, energy must be transferred to it from the signal source. Because energy is drawn from the source, the very act of measurement alters the quantity to be determined. In order to accomplish a measurement successfully, one must minimize the energy drawn from the source or the measurement will have little meaning. The converse of this notion is that without energy transfer, no measurement can be obtained.

The objective of any measurement is to obtain the most representative value $\bar{x}$ for the item measured along with a determination of its uncertainty or precision w_x. In this regard one must understand what a measurement is and how to properly select and/or design the component transducers of the measurement system. One must also understand the dynamic response characteristics of the components of the resulting measurement system in order to properly interpret the readout of the measuring system. The measurement system must be calibrated properly if one is to obtain accurate results. A measure of the repeatability or precision of the measured variable as well as the accuracy of the resulting measurement is important. Unwanted information or "noise" in the output must also be considered when using the measurement system. Until these items are considered, valid data cannot be obtained.

Valid data are defined as those data which support measurement of the most representative value of the desired quantity and its associated precision or uncertainty. When calculated quantities employ measured parameters, one must naturally ask how the precision or uncertainty is propagated to any calculated quantity. Use of appropriate propagation-of-uncertainty equations can yield a final result and its associated precision or uncertainty. Thus the generalized measurement problem requires consideration of the measuring system and its characteristics as well as the statistical analysis necessary to place confidence in the resulting measured quantity. The considerations necessary to accomplish this task are illustrated in Fig. 3.1.

First, a statement of the variables to be measured along with their probable magnitude, frequency, and other pertinent information must be formulated. Next, one brings all the knowledge of fundamentals to the measurement problem at hand. This includes the applicable electronics, engineering mechanics, thermodynamics, heat transfer, economics, etc. One must have an understanding of the variable to be measured if an effective measurement is to be accomplished. For example, if a heat flux is to be determined, one should understand the aspects of heat-energy transfer before attempting to measure entities involved with this process.

Once a complete understanding of the variable to be measured is obtained and the environment in which it is to be measured is understood, one can then consider the necessary characteristics of the components of the measurement system. This would include response, sensitivity, resolution, linearity, and precision. Consideration of these items then leads to selection of the individual instrumentation components, including at least the detector-transducer element, the signal-conditioning element, and a readout element. If the problem is a control situation, a feedback transducer would also be considered. Once the components are selected or specified, they must be coupled to form the generalized measuring system. Coupling considerations to determine the isolation characteristics of the individual transducer must also be made.

Once the components of the generalized measurement system are designed (specified), one can consider the calibration technique necessary to ensure accuracy of the measuring system.

Energy can be transferred into the measuring system by coupling means not at the input ports of the transducer. Thus all measuring systems interact with their environment, so that some unwanted signals are always present in the measuring system. Such "noise" problems must be considered and either eliminated, minimized, or reduced to an acceptable level.

If proper technique has been used to measure the variable of interest, then one has accomplished what is called a *valid measurement*. Considerations of probability and statistics then can result in determination of the precision or uncertainty of the measurement. If, in addition, calculations of dependent variables are to be made from the measured variables, one must consider how the uncertainty in the measured variables propagates to the calculated quantity. Appropriate propagation-of-uncertainty equations must be used to accomplish this task.

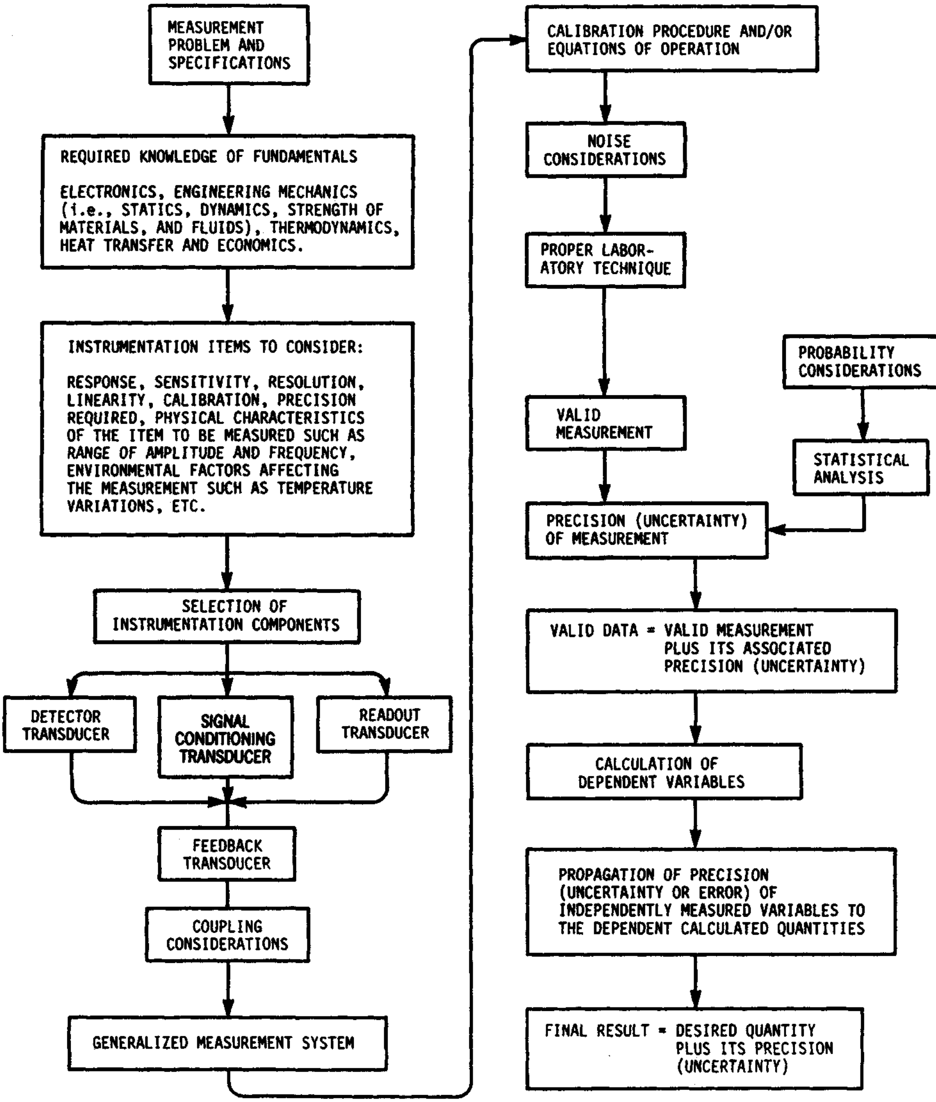


FIGURE 3.1 The generalized measurement task.

3.2 DEFINITION OF MEASUREMENT

A *measurement* is the process of comparing an unknown quantity with a predefined standard. For a measurement to be *quantitative,* the predefined standard must be accurate and reproducible. The standard must also be accepted by international agreement for it to be useful worldwide.

The units of the measured variable determine the standard to be used in the comparison process. The particular standard used determines the accuracy of the measured variable. The measurement may be accomplished by direct comparison with the defined standard or by use of an intermediate reference or calibrated system. The intermediate reference or calibrated system results in a less accurate measurement but is usually the only practical way of accomplishing the measurement or comparison process. Thus the factors limiting any measurement are the accuracy of the unit involved and its availability to the comparison process through reference either to the standard or to the calibrated system.

3.3 STANDARDS OF MEASUREMENT

The defined standards which currently exist are a result of historical development, current practice, and international agreement. The *Système International d'Unités* (or SI system) is an example of such a system that has been developed through international agreement and subscribed to by the standard laboratories throughout the world, including the National Institute of Standards and Technology of the United States.

The SI system of units consists of seven base units, two supplemental units, a series of derived units consistent with the base and supplementary units, and a series of prefixes for the formation of multiples and submultiples of the various units ([3.1], [3.2]).

The important aspect of establishing a standard is that it must be defined in terms of a physical object or device which can be established with the greatest accuracy by the measuring instruments available. The standard or base unit for measuring any physical entity should also be defined in terms of a physical object or phenomenon which can be reproduced in any laboratory in the world.

Of the seven standards, three are arbitrarily selected and thereafter regarded as fundamental units, and the others are independently defined units. The fundamental units are taken as mass, length, and time, with the idea that all other mechanical parameters can be derived from these three. These fundamental units were natural selections because in the physical world one usually weighs, determines dimensions, or times various intervals. Electrical parameters require the additional specification of current. The independently defined units are temperature, electric current, the amount of a substance, and luminous intensity. The definition of each of the seven basic units follows.

At the time of the French Revolution, the unit of *length,* called a *meter* (m), was defined as one ten-millionth of the distance from the earth's equator to the earth's pole along the longitudinal meridian passing through Paris, France. This standard was changed to the length of a standard platinum-iridium bar when it was discovered that the bar's length could be assessed more accurately (to eight significant digits) than the meridian. Today the standard meter is defined to be the length equal to 1 650 763.73 wavelengths in a vacuum of the orange-red line of krypton isotope 86.

The unit of *mass,* called a *kilogram* (kg), was originally defined as the mass of a cubic decimeter of water. The standard today is a cylinder of platinum-iridium alloy kept by the International Bureau of Weights and Measures in Paris. A duplicate with the U.S. National Bureau of Standards serves as the mass standard for the United States. This is the sole base unit still defined by an artifact.

Force is taken as a derived unit from Newton's second law. In the SI system, the unit of *force* is the *newton* (N), which is defined as that force which would give a kilogram mass an acceleration of one meter per second per second.

The unit interval of *time,* called a *second,* is defined as the duration of 9 192 631 770 cycles of the radiation associated with a specified transition of the cesium 133 atom.

The unit of *current,* called the *ampere* (A), is defined as that current flowing in two parallel conductors of infinite length spaced one meter apart and producing a force of 2×10^{-7} N per meter of length between the conductors.

The unit of *luminous intensity,* called the *candela,* is defined as the luminous intensity of one six-hundred-thousandth of a square meter of a radiating cavity at the temperature of freezing platinum (2042 K) under a pressure of 101 325 N/m^2.

The *mole* is the *amount of substance* of a system which contains as many elementary entities as there are carbon atoms in 0.012 kg of carbon 12.

Unlike the other standards, temperature is more difficult to define because it is a measure of the internal energy of a substance, which cannot be measured directly but only by relative comparison using a third body or substance which has an observable property that changes directly with temperature. The comparison is made by means of a device called a *thermometer,* whose scale is based on the *practical international temperature scale,* which is made to agree as closely as possible with the theoretical thermodynamic scale of temperature. The *thermodynamic scale of temperature* is based on the reversible Carnot heat engine and is an ideal temperature scale which does not depend on the thermometric properties of the substance or object used to measure the temperature.

The practical temperature scale currently used is based on various fixed temperature points along the scale as well as interpolation equations between the fixed temperature points. The devices to be used between the fixed temperature points are also specified between certain fixed points on the scale. See Ref. [3.3] for a more complete discussion of the fixed points used for the standards defining the practical scale of temperature.

3.4 THE MEASURING SYSTEM

A measuring system is made up of devices called *transducers.* A transducer is defined as an energy-conversion device [3.4]. A configuration of a generalized measuring system is illustrated in Fig. 3.2.

The purpose of the detector transducer in the generalized system is to sense the quantity of interest and to transform this information (energy) into a form that will be acceptable by the signal-conditioning transducer. Similarly, the purpose of the signal-conditioning transducer is to accept the signal from the detector transducer and to modify this signal in any way required so that it will be acceptable to the readout transducer. For example, the signal-conditioning transducer may be an amplifier, an integrator, a differentiator, or a filter.

The purpose of the readout transducer is to accept the signal from the signal-conditioning transducer and to present an interpretable output. This output may be in the form of an indicated reading (e.g., from the dial of a pressure gauge), or it may be in the form of a strip-chart recording, or the output signal may be passed to either a digital processor or a controller. With a control situation, the signal transmitted to the controller is compared with a desired operating point or set point. This comparison dictates whether or not the feedback signal is propagated through the feedback transducer to control the source from which the original signal was measured.

An *active transducer* transforms energy between its input and output without the aid of an auxiliary energy source. Common examples are thermocouples and piezoelectric crystals. A *passive transducer* requires an auxiliary energy source (AES) to

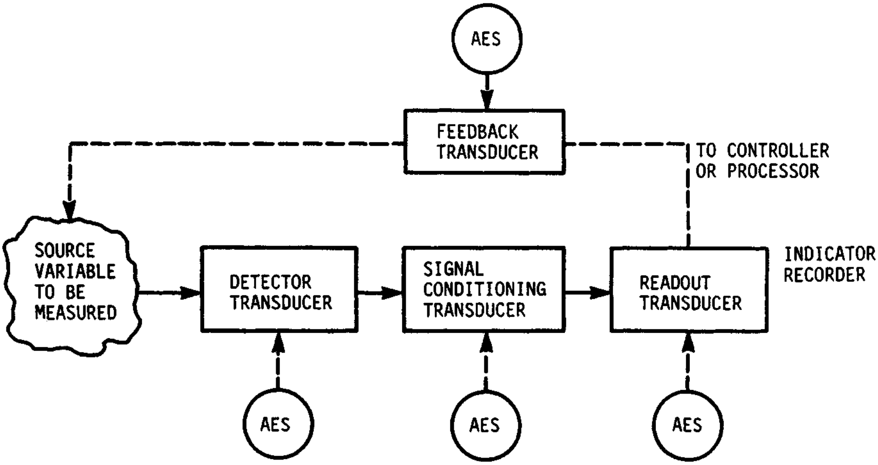


FIGURE 3.2 The generalized measurement system. AES indicates auxiliary energy source, dashed line indicates that the item may not be needed.

carry the input signal through to the output. Measuring systems using passive transducers for the detector element are sometimes called *carrier systems.* Examples of transducers requiring such an auxiliary energy source are impedance-based transducers such as strain gauges, resistance thermometers, and differential transformers. All impedance-based transducers require auxiliary energy to carry the information from the input to the output and are therefore passive transducers.

The components which make up a measuring system can be illustrated with the ordinary thermometer, as shown in Fig. 3.3. The thermometric bulb is the detector or sensing transducer. As heat energy is transferred into the thermometric bulb, the thermometric fluid (for example, mercury or alcohol) expands into the capillary tube of the thermometer. However, the small bore of the capillary tube provides a signal-conditioning transducer (in this case an amplifier) which allows the expansion of the thermometric fluid to be amplified or magnified. The readout in this case is the comparison of the length of the filament of thermometric fluid in the capillary tube with the temperature scale etched on the stem of the thermometer.

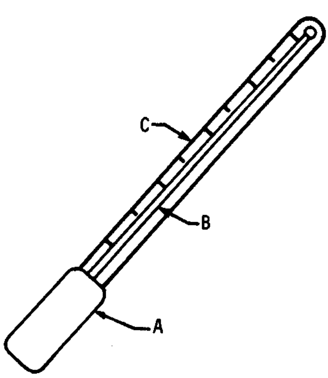


FIGURE 3.3 Components of a simple measuring system. A, detector transducer (thermometer bulb with thermometric fluid); B, signal conditioning stage (amplifier); C, readout stage (indicator).

Another example of an element of a measuring system is the Bourdon-tube pressure gauge. As pressure is applied to the Bourdon tube (a curved tube of elliptical cross section), the curved tube tends to straighten out. A mechanical linkage attached to the end of the Bourdon tube engages a gear of pinion, which in turn is attached to an indicator needle. As the Bourdon tube straightens, the mechanical linkage to the gear on the indicator needle moves, causing the gear and indicating needle to rotate, giving an indication of a change in pressure on the dial of the gauge. The magnitude of the change in pressure is indicated by a pressure scale marked on the face of the pressure gauge.

The accuracy of either the temperature measurement or the pressure measurement previously indicated depends on how accurately each measuring instrument is calibrated. The values on the readout scales of the devices can be determined by means of comparison (calibration) of the measuring device with a predefined standard or by a reference system which in turn has been calibrated in relation to the defined standard.

3.5 CALIBRATION

The *process of calibration* is comparison of the reading or output of a measuring system to the value of known inputs to the measuring system. A complete calibration of a measuring system would consist of comparing the output of the system to known input values over the complete range of operation of the measuring device. For example, the calibration of pressure gauges is often accomplished by means of a device called a *dead-weight tester* where known pressures are applied to the input of the pressure gauge and the output reading of the pressure gauge is compared to the known input over the complete operating range of the gauge.

The type of calibration signal should simulate as nearly as possible the type of input signal to be measured. A measuring system to be used for measurement of dynamic signals should be calibrated using known dynamic input signals. Static, or level, calibration signals are not proper for calibration of a dynamic measurement system because the natural dynamic characteristics of the measurement system would not be accounted for with such a calibration. A typical calibration curve for a general transducer is depicted in Fig. 3.4. It might be noted that the sensitivity of the measuring system can be obtained from the calibration curve at any level of the input signal by noting the relative change in the output signal due to the relative change in the input signal at the operating point.

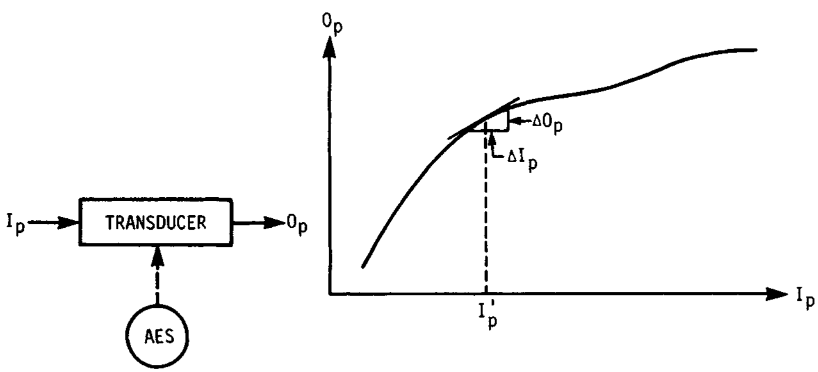


FIGURE 3.4 Typical calibration curve. Sensitivity at $I_p' = (\Delta O_p/\Delta I_p)$.

3.6 DESIGN OF THE MEASURING SYSTEM

The design of a measuring system consists of selection or specification of the transducers necessary to accomplish the detection, transmission, and indication of the desired variable to be measured. The transducers must be connected to yield an interpretable output so that either an individual has an indication or recording of the information or a controller or processor can effectively use the information at the output of the measuring system. To ensure that the measuring system will perform the measurement of the specified variable with the fidelity and accuracy required of the test, the *sensitivity, resolution, range,* and *response* of the system must be known. In order to determine these items for the measurement system, the individual transducer characteristics and the loading effect between the individual transducers in the measuring system must be known. Thus by knowing individual transducer characteristics, the system characteristics can be predicted. If the individual transducer characteristics are not known, one must resort to testing the complete measuring system in order to determine the desired characteristics.

The system characteristics depend on the mathematical order (for example, first-order, second-order, etc.) of the system as well as the nature of the input signal. If the measuring system is a first-order system, its response will be significantly different from that of a measuring system that can be characterized as a second-order system. Furthermore, the response of an individual measuring system of any order will be dependent on the type of input signal. For example, the response characteristics of either a first- or second-order system would be different for a step input signal and a sinusoidal input signal.

3.6.1 Energy Considerations

In order for a measurement of any item to be accomplished, energy must move from a source to the detector-transducer element. Correspondingly, energy must flow from the detector-transducer element to the signal-conditioning device, and energy must flow from the signal-conditioning device to the readout device in order for the measuring system to function to provide a measurement of any variable. Energy can be viewed as having intensive and extensive or primary and secondary components. One can take the primary component of energy as the quantity that one desires to detect or measure. However, the primary quantity is impossible to detect unless the secondary component of energy accompanies the primary component. Thus a force cannot be measured without an accompanying displacement, or a pressure cannot be measured without a corresponding volume change. Note that the units of the primary component of energy multiplied by the units of the secondary component of energy yield units of energy or power (an energy rate). Figure 3.5 illustrates both the active and passive types of transducers with associated components of energy at the input and output terminals of transducers. In Fig. 3.5 the primary component of energy I_p is the quantity that one desires to sense at the input to the transducer. A secondary component I_s accompanies the primary component, and energy must be transferred before a measurement can be accomplished. This means that pressure changes I_p cannot be measured unless a corresponding volume change I_s occurs. Likewise, voltage change I_p cannot be measured unless charges I_s are developed, and force change I_p cannot be measured unless a length change I_s occurs. Thus the units of the product I_pI_s must always be units of energy or power (energy rate). Some important transducer characteristics can now be defined in terms of the energy

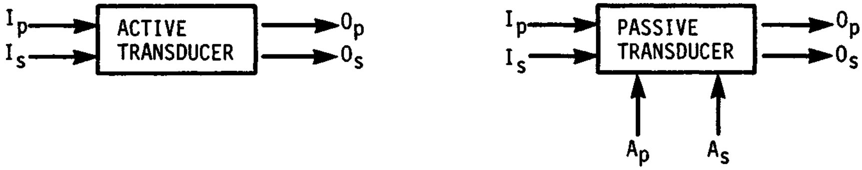


FIGURE 3.5 Energy components for active and passive transducers.

components shown in Fig. 3.5. These characteristics may have both magnitude and direction, so that generally the characteristics are complicated in mathematical nature. A more complete discussion of the following characteristics is contained in Stein [3–4].

3.6.2 Transducer Characteristics

Acceptance ratio of a transducer is defined in Eq. (3.1) as the ratio of the change in the primary component of energy at the transducer input to the change in the secondary component at the transducer input. It is similar to an input impedance for a transducer with electric energy at its input:

$$A = \frac{\Delta I_p}{\Delta I_s} \tag{3.1}$$

Emission ratio of a transducer is defined in Eq. (3.2) as the ratio of the change in the primary component of energy at the transducer output to the change in the secondary component of energy at the transducer output. This is similar to output impedance for a transducer with electric energy at its output:

$$E = \frac{\Delta O_p}{\Delta O_s} \tag{3.2}$$

Transfer ratio is defined in Eq. (3.3) as the ratio of the change in the primary component of energy at the transducer output to the change in the primary component of energy at the transducer input:

$$T = \frac{\Delta O_p}{\Delta I_p} \tag{3.3}$$

Several different types of transfer ratios may be defined which involve any output component of energy with any input component of energy. However, the main transfer ratio involves the primary component of energy at the output and the primary component of energy at the input. The main transfer ratio is similar to the *transfer function,* which is defined as that function describing the mathematical operation that the transducer performs on the input signal to yield the output signal at some operating point. The transfer ratio at a given operating point or level of input signal is also the *sensitivity* of the transducer at that operating point.

When two transducers are connected, they will interact, and energy will be transferred from the source, or first, transducer to the second transducer. When the transfer of energy from the source transducer is zero, it is said to be *isolated* or *unloaded.*

A measure of isolation (or loading) is determined by the *isolation ratio,* which is defined by

$$I = \frac{O_{p,a}}{O_{p,i}} = \frac{O_{p,L}}{O_{p,NL}} = \frac{A}{A + |E_s|} \tag{3.4}$$

where a means actual; i, ideal; L, loaded; and NL, no load.

When the *emission ratio* E_s from the source transducer is zero, the isolation ratio becomes unity and the transducers are isolated. The definition of an *infinite source* or a *pure source* is one that has an emission ratio of zero. The concept of the emission ratio approaching zero is that for a fixed value of the output primary component of energy O_p, the secondary component of energy O_s must be allowed to be as large as is required to maintain the level of O_p at a fixed value. For example, a pure voltage source of 10 V (O_p) must be capable of supplying any number (this may approach infinity) of charges (O_s) in order to maintain a voltage level of 10 V. Likewise, the pure source of force (O_p) must be capable of undergoing any displacement (O_s) required in order to maintain the force level at a fixed value.

Example 1. The transfer ratio (measuring-system sensitivity) of the measuring system shown in Fig. 3.6 is to be determined in terms of the individual transducer transfer ratios and the isolation ratios between the transducers.

Solution

$$T = \frac{O_3}{I_1} = \frac{O_3}{O_{2,L}} \frac{O_{2,L}}{O_{2,NL}} \frac{O_{2,NL}}{O_{1,L}} \frac{O_{1,L}}{O_{1,NL}} \frac{O_{1,L}}{I_1} = T_{32} I_3 T_{21} I_2 T_1$$

$$= \text{(product of transfer ratios) (product of isolation ratios)}$$

3.6.3 Sensitivity

The *sensitivity* is defined as the change in the output signal relative to the change in the input signal at an operating point k. Sensitivity S is given by

$$S = \lim_{\Delta I_p \to 0} \left(\frac{\Delta O_p}{\Delta I_p} \right)_{I_p = k} = \left(\frac{dO_p}{dI_p} \right)_k \tag{3.5}$$

3.6.4 Resolution

The *resolution* of a measuring system is defined as the smallest change in the input signal that will yield an interpretable change in the output of the measuring system at some operating point. Resolution R is given by

$$R = \Delta I_{p,\min} = \frac{\Delta O_{p,\min}}{S} \tag{3.6}$$

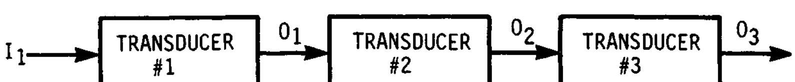


FIGURE 3.6 Measuring-system sensitivity.

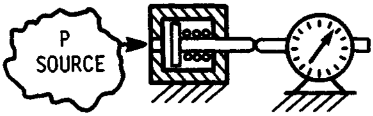


FIGURE 3.7 Pressure transducer in the form of a spring-loaded piston and a dial indicator.

It can be determined by taking the smallest change in the output signal which would be interpretable (as decided by the observer) and dividing by the sensitivity at that operating point.

Example 2. A pressure transducer is to be made from a spring-loaded piston in a cylinder and a dial indicator, as shown in Fig. 3.7. Known information concerning each element is also listed below:

Pneumatic cylinder

- Spring deflection factor = 14.28 lbf/in = K
- Cylinder bore = 1 in
- Piston stroke = ½ in

Dial indicator

- Spring deflector factor = 1.22 lbf/in = k
- Maximum stroke of plunger = 0.440 in
- Indicator dial has 100 equal divisions per 360°
- Each dial division represents a plunger deflection of 0.001 in

The following items are determined:

1. Block diagram of measuring system showing all components of energy (see Fig. 3.8)
2. Acceptance ratio of pneumatic cylinder:

$$A_{PC} = \frac{\Delta I_p}{\Delta I_s} = \frac{P}{V} = \frac{F/A}{AL} = \frac{K}{A^2} = \frac{14.28(16)}{\pi^2} = 23.1 \text{ psi/in}^2$$

3. Emission ratio of pneumatic cylinder:

$$E_{PC} = \frac{\Delta O_p}{\Delta O_s} = \frac{L}{F} = \frac{1}{K} = \frac{1}{14.28} = 0.070 \text{ in/lbf}$$

4. Transfer ratio of pneumatic cylinder:

$$T_{PC} = \frac{\Delta O_p}{\Delta I_p} = \frac{L}{P} = \frac{LA}{F} = \frac{A}{K} = \frac{\pi}{4(14.28)} = 0.055 \text{ in/psi}$$

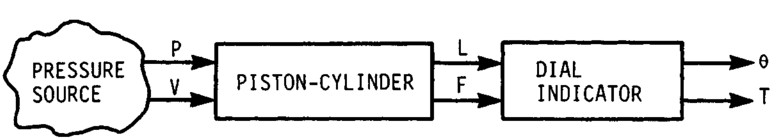


FIGURE 3.8 Pressure-transducer block diagram.

5. Acceptance ratio of dial indicator:

$$A_{DI} = \frac{\Delta I_p}{\Delta I_s} = \frac{L}{F} = \frac{1}{k} = \frac{1}{1.22} = 0.82 \text{ in/lbf}$$

6. Transfer ratio of dial indicator:

$$T_{DI} = \frac{\Delta O_p}{\Delta I_p} = \frac{\theta}{L} = (3.6° \text{ per division})/(0.001 \text{ in per division})$$
$$= 3600°/\text{in (or 1000 divisions/in)}$$

7. Isolation ratio between pneumatic cylinder and dial indicator:

$$I = \frac{A_{DI}}{A_{DI} + E_{PC}} = \frac{1/k}{1/k + 1/K} = \frac{0.82}{0.82 + 0.07} = 0.921$$

8. System sensitivity in dial divisions per psi:

$$S = \frac{\text{output}}{\text{input}} = \frac{DI \text{ output}}{DI \text{ input}} \times \frac{DI \text{ input}}{PC \text{ output}} \times \frac{PC \text{ output}}{PC \text{ input}}$$

$$= T_{DI} I T_{PC} = 0.055(0.921)(1000) = 50.7 \text{ divisions/psi}$$

9. Maximum pressure that the measuring system can sense:

$$\text{Maximum input} = \frac{\text{input}}{\text{output}} \times \text{maximum output} = \frac{1}{S}(440 \text{ dial divisions}) = 8.7 \text{ psi}$$

10. Resolution of the measuring system in psi:

$$\text{Minimum input} = \frac{\text{input}}{\text{output}} \times \text{minimum readable output} = \frac{1}{S}(1 \text{ dial division}) = 0.02 \text{ psi}$$

3.6.5 Response

When time-varying signals are to be measured, the dynamic response of the measuring system is of crucial importance. The components of the measuring system must be selected and/or designed such that they can respond to the time-varying input signals in such a manner that the input information is not lost in the measurement process. Several measures of response are important to know if one is to evaluate a measuring system's ability to detect and reproduce all the information in the input signal. Some measures of response involve time alone, whereas other measures of response are more involved. Various measures of response are defined in the following paragraphs.

Amplitude response of a transducer is defined as the ability to treat all input amplitudes uniformly [3.5]. The typical amplitude-response curve determined for either an individual transducer or a complete measuring system is depicted in Fig. 3.9.

A typical amplitude-response specification is as follows:

$$\left|\frac{O_p}{I_p}\right| = M \pm T \qquad I_{p,\min} < I_p < I_{p,\max} \tag{3.7}$$

The amplitude-response specification includes a nominal magnitude ratio M between output and input of the transducer measuring system along with an allowable tolerance T and a specification of the range of the magnitude of the primary input variable I_p over which the amplitude ratio and tolerance are valid.

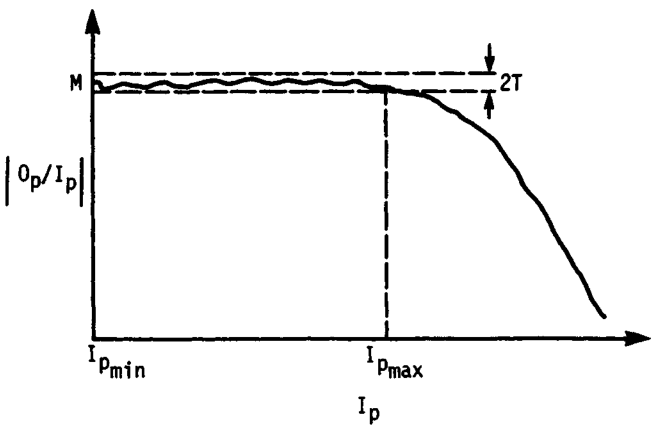


FIGURE 3.9 Typical amplitude-response characteristic.

Frequency response can be defined as the ability of a transducer to treat all input frequencies uniformly [3.5] and can be specified by a frequency-response curve such as that shown in Fig. 3.10. A typical frequency-response specification would be the nominal magnitude ratio M of output to input signals plus or minus some allowable tolerance T specified over a frequency range from the low-frequency limit f_L to the high-frequency limit f_H as follows:

$$\left|\frac{O_p}{I_p}\right| = M \pm T \qquad f_L < f < f_H \tag{3.8}$$

It is the usual practice to use the decibel (dB) rather than the actual magnitude ratio for the ordinate of the frequency-response curve. The decibel, as defined in Eq. (3.9), is used in transducers and measuring systems in specifying frequency response:

$$\text{Decibel} = 20 \log_{10} \frac{O_p}{I_p} \tag{3.9}$$

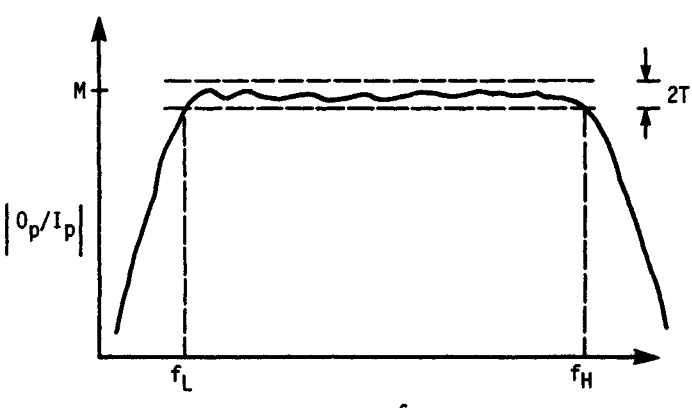


FIGURE 3.10 Typical frequency-response characteristic.

The decibel scale allows large gains or attenuations to be expressed as relatively small numbers.

Phase response can be defined as the ability of a transducer to treat all input-phase relations uniformly [3.5]. For a pure sine wave, the phase shift would be a constant angle or a constant time delay between input and output signals. Such a constant phase shift or time delay would not affect the waveform shape or amplitude determination when viewing at least one complete cycle of the waveform. For complex input waveforms, each harmonic in the waveform may be treated slightly differently in the measuring system, resulting in what is known as *phase distortion,* as illustrated in Ref. [3.5].

Response times are valid measures of response of transducers and measuring systems. An understanding of the response-time specifications requires that the mathematical order of the system be known and that the type of input signal or forcing function be specified.

Rise time of a transducer or measuring system is defined for any order system subjected to a step input. The *rise time* is defined as that time for the transducer or measuring system to respond from 10 to 90 percent of the step-input amplitude and is depicted in Fig. 3.11.

Delay time is another response time which is defined for any order system subjected to a step input. The *delay time* is defined to be that time for the transducer or measuring system to respond from 0 to 50 percent of the step-input amplitude and is depicted in Fig. 3.11.

Time constant is specifically defined for a first-order system subjected to a step input. The *time constant* τ is defined as the time for the transducer or measuring system to respond to 63.2 percent (or $1 - e^{-1}$) of the step-input amplitude. The time constant is specifically illustrated in Fig. 3.12, where the response x of the first-order system to step input x_s is known to be exponential as follows:

$$x = x_s(1 - e^{-t/\tau}) \tag{3.10}$$

When the time t is equal to the time constant τ, the first-order system has responded to 63.2 percent of the step-input amplitude. In a time span equivalent to

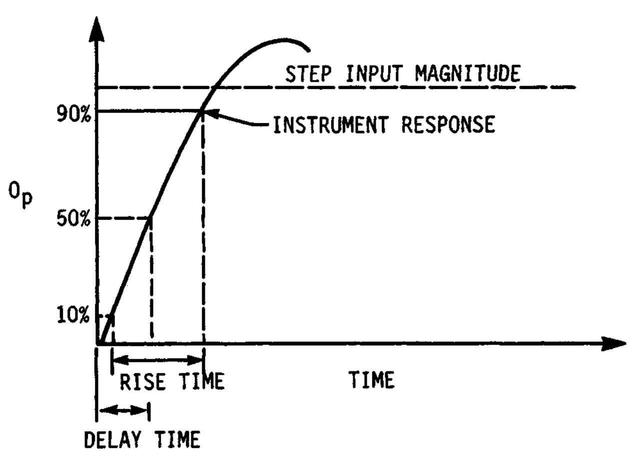


FIGURE 3.11 Rise time and delay time used as response times.

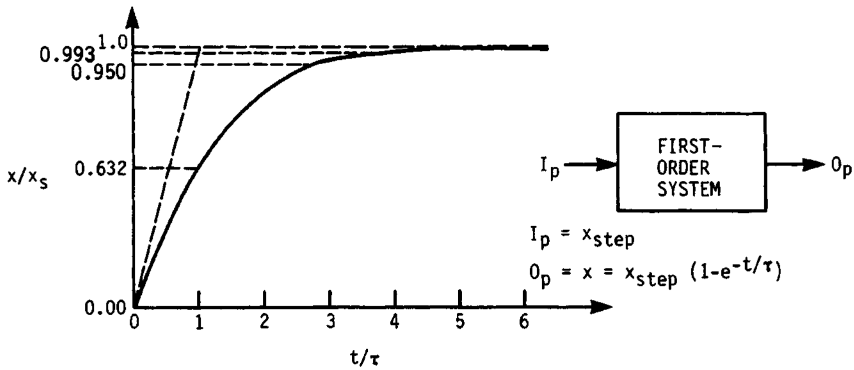


FIGURE 3.12 Response of a first-order system to a step input.

3 time constants, the system has responded to 95.0 percent of the step-input amplitude, and in a time span of 5 time constants, the system has responded to 99.3 percent of the step-input amplitude. Thus for a first-order system subjected to a step input to yield a correct reading of the input variable, one must wait a time period of at least 5 time constants in order for the first-order system to respond sufficiently to give a correct indication of the measured variable.

Transducer Dynamics. Because of the time delay or phase shift a transducer or measuring system may have, one must be very careful to ensure that the measuring system can respond adequately if the input signal to the measuring system is varying with time. If the time response of the measuring system is inadequate, it may never read the correct value. Thus if one believes the output indication of the measuring system to be a reproduction of the actual value of the input (measured) variable without understanding the dynamics of how the measuring system is responding to the input signal, a crucial error can be made.

In order to understand dynamic response, one must recognize that the components of the measuring system have natural physical characteristics and that the measuring system will tend to respond according to these natural characteristics when perturbed by any external disturbance. In addition, the input signal supplied to a transducer or measuring system provides a forcing function for that transducer or measuring system. The equation of operation of a transducer is a differential equation whose order is defined as the order of the system. The response of the system is determined by solving this differential equation of operation according to the type of input signal (forcing function) supplied to the system. If the measuring system is modeled as a linear system, the differential equation of operation will be ordinary and linear with constant coefficients. This is the type of differential equation that can be solved by well-known techniques. The nature of the solution depends on the nature of the forcing function as well as the nature of the physical components of the system. For example, the thermometric element of the temperature-measuring device can be modeled as shown in Fig. 3.13. For this model,

$$q_{\text{in}} = q_{\text{lost}} + q_{\text{stored}} = \text{rate of heat energy entering control region}$$

and

$$q_{\text{in}} = hA(T_\infty - T)$$
$$q_{\text{lost}} = 0 \text{ (assumed)}$$
$$q_{\text{stored}} = \rho c v \frac{dT}{dt}$$

where A = surface area
h = surface-film coefficient of convective heat transfer
ρ = density of thermometric element
c = specific heat capacity of thermometric element
T = temperature of thermometric element
t = time

The resulting equation of the operation is given as follows for the step input $x_s = T_\infty - T_0$:

$$T - T_0 = (T_\infty - T_0)(1 - e^{-t/\tau}) \tag{3.11}$$

where $\tau = \rho vc/hA$. The response $x = T - T_0$ is shown in Fig. 3.12.

Another example of a first-order system is the electric circuit composed of resistance and capacitive elements or the so-called RC circuit. Masses falling in viscous media also follow a similar exponential characteristic.

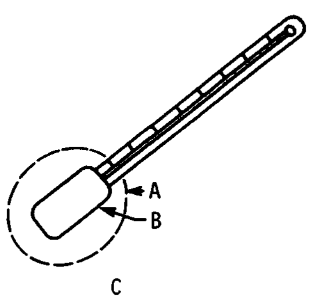


FIGURE 3.13 Thermometric element modeled as a first-order system. A, control region; B, thermometric element at temperature T; C, environment at temperature T_∞.

If the system is characterized by a second-order linear ordinary differential equation, the solution becomes more complex than that for the first-order system. The system behavior depends on the amount of friction or damping in the system. For example, the meter movement of a galvanometer or D'Arsonval movement shown in Fig. 3.14 such as exists in many electrical meters can be modeled as shown in Fig. 3.15. Applying first principles to this model yields the equation of motion

$$\Sigma T = J\ddot{\theta} = T(t) - T_s - T_f$$

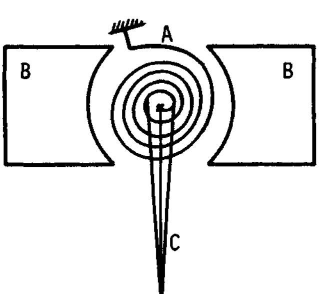


FIGURE 3.14 D'Arsonval movement. A, spring-retained armature; B, field magnets; C, indicating needle.

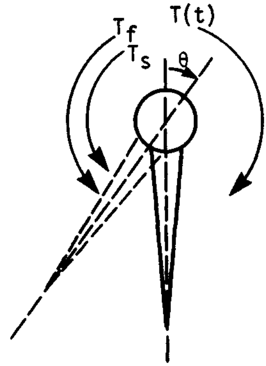


FIGURE 3.15 Torques applied to the D'Arsonval movement.

where $T_s = k\theta$ for torsional damping
$T_f = \sigma\dot{\theta}$ for viscous friction
$T(t)$ = driving or forcing function

Then
$$J\ddot{\theta} + \sigma\dot{\theta} + k\theta = T(t)$$

or
$$\ddot{\theta} + 2\gamma\omega_n\dot{\theta} + \omega_n^2\theta = \frac{T(t)}{J} \tag{3.12}$$

where $\omega_n = \sqrt{k/J}$ = natural undamped frequency
$\omega_d = \omega_n\sqrt{1-\gamma^2}$ = natural damped frequency
$\omega_p = \omega_n\sqrt{1-2\gamma^2}$ = frequency at peak of frequency response curve
$\gamma = \sigma/\sigma_c$ = damping ratio
$\sigma_c = \sqrt{4kJ}$ = critical value of damping
= lowest value of damping where no natural oscillation of system occurs

If the damping is modeled as viscous friction, the possible solutions to the equation of motion are given by Eqs. (3.13), (3.14), and (3.15) for the step input. The underdamped solution of Eq. (3.12) is shown in Fig. 3.16.

For $\sigma < 1$ (underdamped),

$$\frac{x}{x_s} = 1 - \{1-\gamma^2\}^{-1/2} \exp(-\gamma\omega_n t) \sin(\omega_d t + \phi)$$

$$\phi = \tan^{-1}\sqrt{\frac{1-\gamma^2}{\gamma}} \tag{3.13}$$

For $\sigma = 1$ (critical damping),

$$\frac{x}{x_s} = 1 - (1 + \omega_n t)\exp(-\omega_n t) \tag{3.14}$$

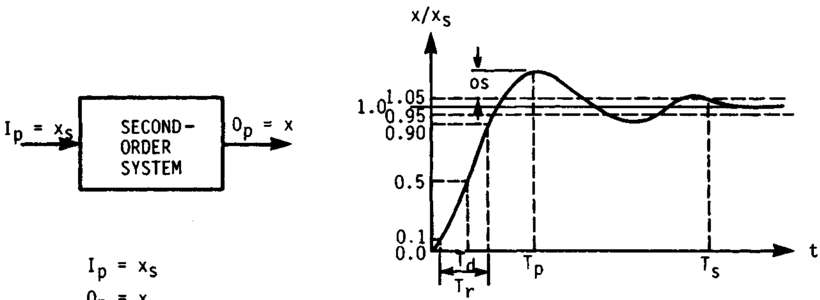


FIGURE 3.16 Response of a second-order system to a step input.

For $\sigma > 1$ (overdamped),

$$\frac{x}{x_s} = 1 - \left(\frac{\beta}{\beta - 1}\right)\left[\exp\left(\frac{-\omega_n t}{\sqrt{\beta}}\right) - \frac{1}{\beta}\exp\left(-\sqrt{\beta}\,\omega_n t\right)\right]$$

$$\beta = \frac{\gamma + \sqrt{\gamma^2 - 1}}{\gamma - \sqrt{\gamma^2 - 1}} \tag{3.15}$$

If the system is underdamped, the response of the transducer or measuring system overshoots the step-input magnitude and the corresponding oscillation occurs with a first-order decay. This type of response leads to additional response specifications which may be used by transducer manufacturers. These specifications include *overshoot OS, peak time* T_p, *settling time* T_s, *rise time* T_r, and *delay time* T_d as depicted in Fig. 3.16. If the viscous damping is at the critical value, the measuring system responds up to the step-input magnitude only after a very long period of time. If the damping is more than critical, the response of the measuring system never reaches a magnitude equivalent to the step input. Measuring-system components following a second-order behavior are normally designed and/or selected such that the damping is less than critical. With underdamping the second-order system responds with some time delay and a characteristic phase shift.

If the natural response characteristics of each measuring system are not known or understood, the output reading of the measurement system can be erroneously interpreted. Figure 3.17 illustrates the response of a first-order system to a square-wave input. Note that the system with inadequate time response never yields a valid indication of the magnitude of the step input. Figure 3.18 illustrates a first-order system with time constant adequate ($\tau << 1/f$) to yield a valid indication of step-input magnitude. Figure 3.19 illustrates the response of an underdamped second-order system to a square-wave input. A valid indication of the step-input magnitude is obtained after the settling time has occurred.

If the input forcing function is not a step input but a sinusoidal function instead, the corresponding differential equations of motion to the first- and second-order systems are given in Eqs. (3.16) and (3.17), respectively:

$$\dot{x} + \frac{x}{\tau} = A\cos\omega_f t \tag{3.16}$$

where A = amplitude of input signal transformed to units of the response variable derivative(s)
ω_f = frequency of input signal (forcing function)
τ = time constant

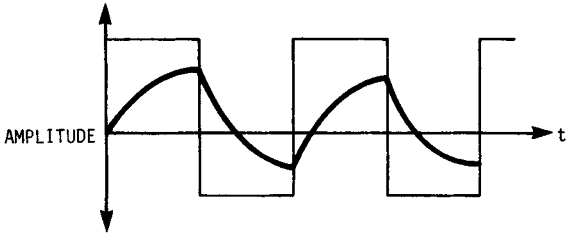


FIGURE 3.17 Response of a first-order system with inadequate response to a square-wave input ($\tau > 1/f$).

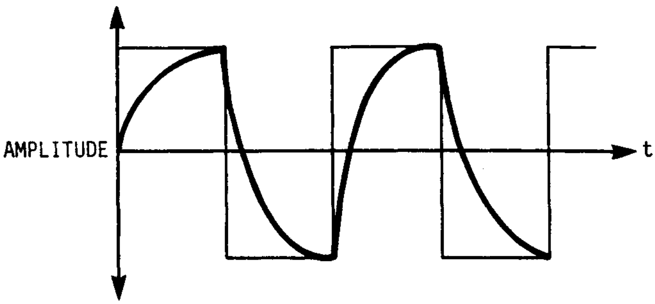


FIGURE 3.18 Response of a first-order system with barely adequate response to a square-wave input ($\tau << 1/f$).

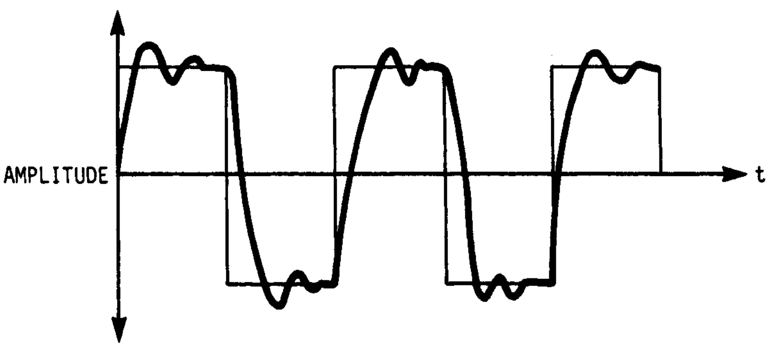


FIGURE 3.19 Response of an underdamped second-order system to a square wave.

$$\ddot{x} + 2\sigma\omega_n\dot{x} + \omega_n^2 x = A \cos \omega_f t \tag{3.17}$$

In addition, the parameters of the steady-state responses of the first- and second-order system are given by Eqs. (3.18) and (3.19), respectively, and are shown in Figs. 3.20 and 3.21. The steady-state solutions are of the form

$$x_{ss} = B \cos (\omega_f t + \phi)$$

where, for the first- and second-order systems, respectively,

$$B_1 = \frac{A}{\sqrt{(\tau\omega_f)^2 + 1}} \qquad \phi_1 = -\tan^{-1} (\tau\omega_f) \tag{3.18}$$

$$B_2 = \frac{A}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + (2\gamma\, \omega_f/\omega_n)^2}} \qquad \phi_2 = -\tan^{-1} \frac{2\gamma\, \omega_f/\omega_n}{1 - (\omega_f/\omega_n)^2} \tag{3.19}$$

From these results it can be noted that both the first- and second-order systems, when responding to sinusoidal input functions, experience a magnitude change and a phase shift in response to the input function.

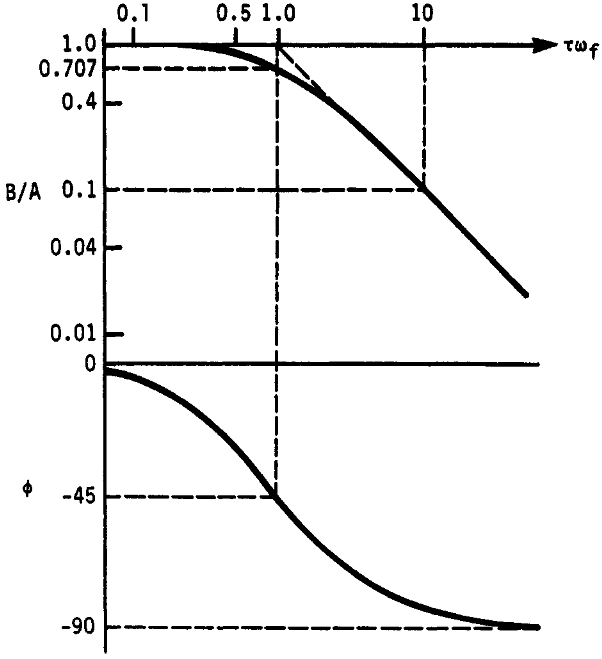


FIGURE 3.20 Frequency and phase response of a first-order system to a sinusoidal input.

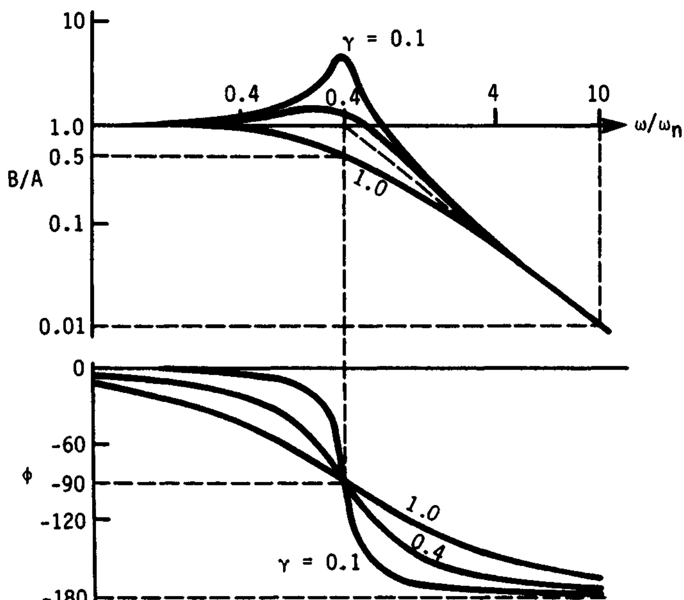


FIGURE 3.21 Frequency and phase response of a second-order system to a sinusoidal input.

Many existing transducers behave according to either a first- or second-order system. One should understand thoroughly how both first- and second-order systems respond to both the step input and sinusoidal input in order to understand how a transducer is likely to respond to such input signals. Table 3.1 is a listing of the steady-state responses of both the first- and second-order systems to a step function, ramp function, impulse function, and sinusoidal function. (See also [3.6] and [3.7].)

Understanding how a transducer might respond to a complex transient waveform can be understood by considering a sinusoidal response of the system, since any complex transient forcing function can be represented by a Fourier series equivalent [3.5]. Consideration of each separate harmonic in the input forcing function would then yield information as to how the measuring system is likely to respond.

Example 3. A thermistor-type temperature sensor is found to behave as a first-order system, and its experimentally determined time constant τ is 0.4 s. The resistance-temperature relation for the thermistor is given as

$$R = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

where β has been experimentally determined to be 4000 K. This temperature sensor is to be used to measure the temperature of a fluid by suddenly immersing the thermistor into the fluid medium.

How long one must wait to ensure that the thermometer reading will be in error by no more than *5 percent of the step change in temperature* is calculated as follows:

$$\begin{aligned}
x &= x_s(1 - e^{-t/\tau}) \\
x &= T - T_0 = 0.95(T_\infty - T_0) \\
x_s &= T_\infty - T_0 \\
\therefore 0.95 &= 1 - e^{-t/0.4} \\
\ln 0.05 &= \frac{-t}{0.4} = -2.9957 \\
\therefore t &= 1.198 \text{ s} = 1.2 \text{ s}
\end{aligned}$$

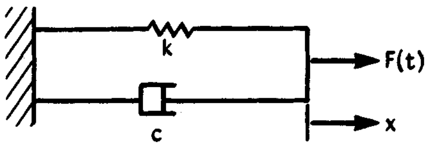
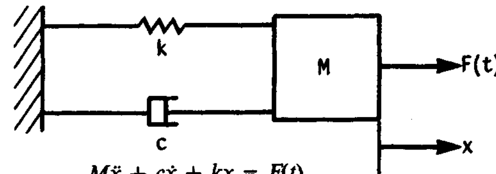
Determine the sensitivity of the thermometer at a temperature of 300 K if the resistance R is 1000 ohms (Ω) at this temperature:

$$\begin{aligned}
S = \left.\frac{dR}{dT}\right|_{op} &= R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]\beta(-1)T^{-2} \\
&= -\frac{R\beta}{T^2} = \frac{1000(4000)}{(300)^2} \\
&= -44.44 \ \Omega/\text{K}
\end{aligned}$$

Determine the resolution of the thermometer if one can observe changes in resistance of 0.50 Ω on a Wheatstone bridge used as a readout device at the temperature of 300 K:

$$R = \frac{\Delta Q_{op}|_{\min}}{S} = \frac{-0.50}{-44.44} = 0.0113 \text{ K}$$

TABLE 3.1 Response of First- and Second-Order Systems to Various Input Signals

First-order system	Second-order system
Equation of Motion	
 $c\dot{x} + kx = F(t)$ $\tau\dot{x} + x = \dfrac{F(t)}{k}$ $\tau = \dfrac{c}{k}$	 $M\ddot{x} + c\dot{x} + kx = F(t)$ $\ddot{x} + 2\gamma\omega_n\dot{x} + \omega_n^2 x = \dfrac{F(t)}{M}$ where $\gamma = \dfrac{c}{c_c}$ $\quad c_c = \sqrt{4kM}$ $\quad \omega_n = \sqrt{\dfrac{k}{M}}$
Step input: $F(t) = F \qquad t > 0$	
$\dfrac{x}{F/k} = 1 - \exp(-t/\tau)$	(a) $\gamma < 1$: $\dfrac{x}{F/k} = 1 - \dfrac{\exp(-\gamma\omega_n t)}{\sqrt{1-\gamma^2}} \sin\left(\sqrt{1-\gamma^2}\,\omega_n t + \phi\right)$ $\phi = \tan^{-1}\dfrac{\sqrt{1-\gamma^2}}{\gamma}$ (b) $\gamma = 1$: $\dfrac{x}{F/k} = 1 - (1 + \omega_n t)\exp(-\omega_n t)$ (c) $\gamma > 1$: $\dfrac{x}{F/k} = 1 - \dfrac{\nu}{\nu - 1}\left[\exp\dfrac{-\omega_n t}{\sqrt{\nu}} - \dfrac{1}{\nu}\exp(-\sqrt{\nu}\,\omega_n t)\right]$ $\nu = \dfrac{\gamma + \sqrt{\gamma^2 - 1}}{\gamma - \sqrt{\gamma^2 - 1}}$

Impulse input: $I = \int_0^t F\,dt \qquad t \to 0$	
$\dfrac{xk\tau}{I} = \exp(-t/\tau)$	(*a*) $\gamma < 1$: $\dfrac{x\sqrt{Mk}}{I} = \dfrac{\exp(-\gamma\omega_n t)}{\sqrt{1-\gamma^2}} \sin(\sqrt{1-\gamma^2}\,\omega_n t)$ (*b*) $\gamma = 1$: $\dfrac{x\sqrt{Mk}}{I} = \omega_n t \exp(-\omega_n t)$ (*c*) $\gamma > 1$: $\dfrac{x\sqrt{Mk}}{I} = \dfrac{\sqrt{\nu}}{\nu - 1} \exp\left(\dfrac{-\omega_n t}{\sqrt{\nu}}\right) - \exp(\sqrt{\nu}\,\omega_n t)$
Ramp input: $F(t) = \beta t$	
$\dfrac{xk}{\beta\tau} = \dfrac{t}{\tau} - [1 - \exp(-t/\tau)]$	(*a*) $\gamma < 1$: $\dfrac{x\omega_n k}{\beta} = \dfrac{1}{\sqrt{1-\gamma^2}} \exp(-\gamma\omega_n t) \sin(\sqrt{1-\gamma^2}\,\omega_n t + \phi) - 2\gamma + \omega_n t$ $\phi = \tan^{-1} \dfrac{\gamma\sqrt{1-\gamma^2}}{\gamma^2 - \frac{1}{2}}$ (*b*) $\gamma = 1$: $\dfrac{x\omega_n k}{\beta} = (2 + \omega_n t)\exp(-\omega_n t) + \omega_n t - 2$ (*c*) $\gamma > 1$: $\dfrac{x\omega_n k}{\beta} = \dfrac{\nu\sqrt{\nu}}{\nu - 1} \exp\left(\dfrac{-\omega_n t}{\sqrt{\nu}}\right) - \dfrac{1}{\nu^2} \exp(-\sqrt{\nu}\,\omega_n t) - \dfrac{\nu + 1}{\sqrt{\nu}} + \omega_n t$

TABLE 3.1 Response of First- and Second-Order Systems to Various Input Signals (*Continued*)

First-order system	Second-order system
	Sinusoidal input: $F(t) = F_0 \cos \Omega t$ or $F(t) =$ (real part of)$F_0 \exp(i\Omega t)$
$\dfrac{x}{(F_0/k)} = \dfrac{\cos(\Omega t + \phi)}{\sqrt{1 + (\Omega\tau)^2}}$ $-\phi = \tan^{-1} \Omega\tau$	$\dfrac{x}{F_0/k} = \dfrac{\cos(\Omega t + \phi)}{\sqrt{(1 - \beta^2)^2 + (2\gamma\beta)^2}}$ $\phi = \tan^{-1} \dfrac{-2\gamma\beta}{1 - \beta^2}$ $\beta = \dfrac{\Omega}{\omega_n}$

The expected response of the thermometer if it were subjected to step changes in temperature between 300 and 500 K in a square-wave fashion and at a frequency of 1.0 hertz (Hz) is shown in Fig. 3.22, where $x = x_s$ (0.7135). Note that the thermistor never responds sufficiently to give an accurate indication of the step-amplitude temperature. However, if the time constant of the thermistor were selected to be less than 0.1 s, the step-amplitude temperature would be indicated in 0.5 s (5 time constants).

Example 4. A strip-chart recorder (oscillograph) has been determined to behave as a second-order system with damping ratio of 0.5 and natural frequency of 60 Hz. At what frequency would the output amplitude of the recorder "peak" even with a constant-amplitude input signal? The frequency may be calculated as follows:

$$\omega_p = \omega_n \sqrt{1 - 2\gamma^2} = 60\sqrt{1 - 2(0.5)^2} = 42.4 \text{ Hz}$$

What is the maximum sine-wave frequency of input signal that would allow no more than 5 percent error in amplitude? See Fig. 3.23. The amplitude factor (AF) is calculated as follows:

$$1.05 = \text{AF} = \frac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + (2\gamma\, \omega_f/\omega_n)^2}} = \frac{1}{\sqrt{1 - z + z^2}}$$

where $z \equiv (\omega_f/\omega_n)^2$. The result is $\omega_{f_{\max}} = 19.2$ Hz.

A complex waveform made up of a fundamental frequency of 10 Hz and 8 harmonics in terms of its Fourier series representation is desired to be recorded. Will the oscillograph described above suffice?

The basic equation is

$$\text{Maximum frequency} = (n + 1)(\text{fundamental}) = 90 \text{ Hz}$$

$$\text{AF} = \frac{1}{\sqrt{[(1 - (90/60)^2]^2 + (90/60)^2}} = 0.51$$

$$\psi = \tan^{-1} \frac{2(0.5)90/60}{1 - (90/60)^2} = -55.2° \qquad \text{(oscillograph will not suffice)}$$

If both the frequency and phase-response characteristics for the oscillograph are given below, show how the input signal to the oscillograph, also given below, will be changed, and give the resulting relation expected:

$$e = 10 + 5.8 \cos 5t + 3.2 \cos 10t + 1.8 \cos 20t$$

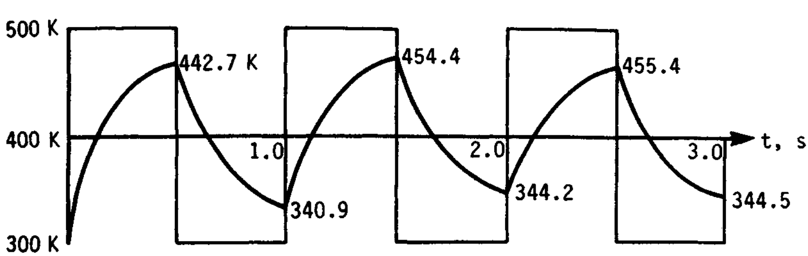


FIGURE 3.22 Thermistor temperature response of Example 3.

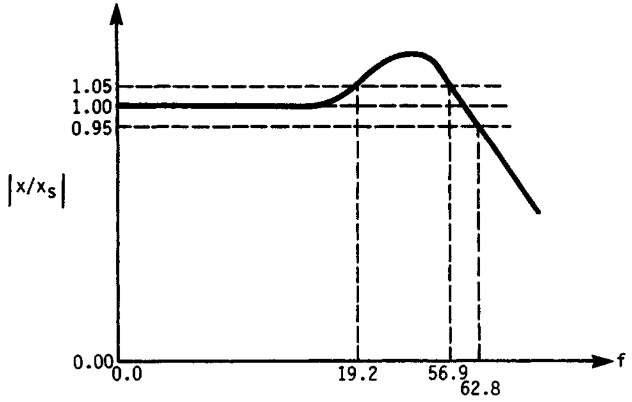


FIGURE 3.23 Frequency response of strip-chart recorder of Example 4.

Input frequency ω, rad/s	Amplitude, V Input	Output	Phase angle (lag), °
0	10.0	10.0	0
5	10.0	10.0	10
10	10.0	10.2	20
15	10.0	10.6	30
20	10.0	11.0	45
25	10.0	12.2	90

It follows that

$$e_o = 10\left(\frac{10}{10}\right) + 5.8\left(\frac{10.0}{10.0}\right)\cos\left(5t - \frac{10\pi}{180}\right) + 3.2\left(\frac{10.2}{10.0}\right)\cos\left(10t - \frac{20\pi}{180}\right)$$

$$+ 1.8\left(\frac{11.0}{10.0}\right)\cos\left(20t - \frac{45\pi}{180}\right)$$

$$= 10 + 5.8\cos(5t - 0.174) + 3.26\cos(10t - 0.349) + 1.98\cos(20t - 0.785)$$

3.7 SELECTED MEASURING-SYSTEM COMPONENTS AND EXAMPLES

3.7.1 Operational Amplifiers

Operational amplifiers [3.8] used in measuring systems have the basic configuration shown in Fig. 3.24. The *operational amplifier* is composed of a high-gain voltage amplifier coupled with both input and feedback impedances. The characteristics of